DISCONTINUITY PRESERVING REGULARIZATION FOR MODELING SLIDING EFFECTS IN MEDICAL IMAGE REGISTRATION

Dan Ruan, Jeffrey A. Fessler[*]

The University of Michigan
Dept. of Electrical Eng. & Computer Sci.

Selim Esedoḡlu[†]

The University of Michigan
Dept. of Mathematics

ABSTRACT

Sliding effects often occur along tissue/organ boundaries. However, most conventional registration techniques either use smooth parametric bases or apply homogeneous smoothness regularization, and fail to address the sliding issue. In this study, we propose a class of discontinuity-preserving regularizers that fit naturally into optimization-based registration. The proposed regularization encourages smooth deformations in most regions, but preserves large discontinuities supported by the data. Variational techniques are used to derive the descending flows. We discuss general conditions on such discontinuity-preserving regularizers, their properties based on an anisotropic filtering interpretation. Preliminary tests with 2D CT data show promising results.

Index Terms— variational methods, image registration, adaptive filters.

1. INTRODUCTION

Medical registration techniques aim to find the coordinate transformation that best matches two images. In general, organ and tissue motions are nonrigid, and demonstrate high degrees of freedom, which makes medical image registration problems ill-posed. Prior knowledge about the underlying physical process is incorporated to address this challenge. In particular, smoothness of the transformation is widely utilized: parametric methods use smooth basis functions (such as B-spline) and optimize over relatively small number of coefficients; fully nonparametric methods such as optical flow and bio-mechanical models with finite elements either build in smoothness constraints or incorporate smoothness regularization in an optimization framework. However, sliding along tissue/organ boundary widely exists, and homogeneous smoothing of the transformation field blurs the estimated transformation across the sliding interface, resulting in undesirable artifacts.

Recently, several studies [1, 2] of joint segmentation and registration have arisen from various disciplines and applications. In these methods, smooth regions and singularity sets (edges) are devised according to image intensity, and registration aims to align each part respectively. The smoothness and discontinuity in the deformation itself is not addressed directly.

In this study, we propose a class of regularization schemes that preserve discontinuities in the deformation field. We provide general analysis on their functional forms, and some desired properties as a consequence. We derive the descending flow for optimization and discuss briefly some implementation issues. A preliminary 2D test with clinical CT data shows promising results.

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2. PROPOSED METHOD

For simplicity, we discuss the 2D case, yet all analysis generalizes to higher dimensions. We denote the source and target images as $f, g : \Omega \subset \Re^2 \rightarrow \Re$ where Ω denotes the region of interest (ROI). We denote the deformation vector field as $W : \Omega \rightarrow \Re^2$ with $W(\mathbf{x}) = [U(\mathbf{x}), V(\mathbf{x})]^T$, *i.e.*, U and V are directional deformation and assumed to be orthogonal (but does not have to align with the image coordinate (x, y)) in general. Regularized registration aims to find

$$\begin{aligned} W^* &= \arg\min_{W\in\Gamma} E(W, f, g) \qquad (1)\\ &= \arg\min_{W\in\Gamma} \{E_d(g, f\circ(I+W)) + \lambda E_r(W)\}, \end{aligned}$$

where Γ is the allowable set of deformations and λ controls the tradeoff between data fidelity and regularization energy. The choice of E_d depends on the image modality and the quantity of interest. We focus here on designing E_r.

To encourage smooth deformations in most of the region of interest (ROI), yet admitting some discontinuities requires a "magnitude" measure of the local change of the deformation field, analogous to the norm of image gradient in intensity domain. The Jacobian of the deformation W at $\mathbf{x}$ is given by:

$$DW(\mathbf{x}) = \begin{bmatrix} U_x & U_y \\ V_x & V_y \end{bmatrix}.$$

We propose to use the Frobenius norm of the matrix $DW(\mathbf{x})$

as the local measure of variation for the deformation field:

$$
\begin{aligned}
|DW|_{\text{Frob}} &= \sqrt{U_x^2 + U_y^2 + V_x^2 + V_y^2} \qquad (2)\\
&= \sqrt{|\nabla U|_2^2 + |\nabla V|_2^2}.
\end{aligned}
$$

This matrix norm is independent of both the image coordinate system $x-y$ and the deformation vector field direction $u-v$. In addition, this measure of "deformation change" introduces coupling among the various directions in the vector fields and reflects the intuition that we observe a "jump" in the deformation field regardless of the specific direction such change occur, unlike the simple coordinate-wise sum used in traditional optical flow regularization [3, 4]. For simplicity, we make matrix Frobenius norm the default notation for $|DW|$ hereafter and drop the subscript.

We consider a class of regularizers with the form:

$$
E_r(W) = \int \phi(|DW|) d\mathbf{x}.
$$

Applying variational analysis, and assuming Neuman boundary conditions, *i.e.*, $\partial_n U = 0$ and $\partial_n V = 0$ on $\partial\Omega$, we derive the descent flow $w_r = (u_r, v_r)$ of E_r to be as follows:

$$
\begin{aligned}
u_r &= \nabla \cdot \big(\frac{\partial}{\partial \nabla u}\phi\big) \qquad (3)\\
&= \nabla \cdot \big(\frac{\phi'(|DW|)}{|DW|}\nabla U\big).
\end{aligned}
$$

The expression for the update flow v_r for V is similar. For simplicity, we define the "influence function" as $\psi(s) \triangleq \phi'(s)/s$.

To design a proper regularization ϕ that results in edge preserving flow, we interpret the process as anisotropic filtering and decompose the effect of the flow into the normal and tangent directions for *each component of the deformation field.* It can be shown that the regularization flow in $u-$ direction is:

$$
\begin{aligned}
u_r = {} & \psi(|DW|)(U_{xx} + U_{yy}) + \qquad (4)\\
& + \tfrac{\phi''(|DW|) - \psi(|DW|}{|DW|^2}(U_x^2 U_{xx} + 2U_x U_y U_{xy} + U_y^2 U_{yy})
\end{aligned}
$$

By convention, we denote the second derivative of U in the tangent (T-) direction and normal (N-) direction as U_{TT} and U_{NN} respectively, with

$$
U_{TT} = T^T \Delta U T = \frac{1}{|\nabla U|}(U_x^2 U_{yy} + U_y^2 U_{xx} - 2U_x U_y U_{xy});
$$

$$
U_{NN} = N^T \Delta U N = \frac{1}{|\nabla U|}(U_x^2 U_{xx} + U_y^2 U_{yy} + 2U_x U_y U_{xy}).
$$

Rearranging the terms in (5) results in:

$$
\begin{aligned}
u_r = {} & \psi(|DW|)U_{TT} + \qquad (5)\\
& + |\nabla U|^2 \{\tfrac{\phi''(|DW|)}{|DW|^2} - \tfrac{\psi(|DW|)}{|DW|^2} + \tfrac{\psi(|DW|)}{|\nabla U|^2}\} U_{NN}
\end{aligned}
$$

For 2D case (higher dimension situations have similar structure):

$$
\frac{\psi(|DW|)}{|\nabla U|^2} - \frac{\psi(|DW|)}{|DW|^2} = \psi(|Dw|)\frac{|\nabla V|^2}{|DW|^2 |\nabla U|^2}.
$$

The coupling between U and V in the flow motivates us to consider the contribution of variation in each deformation direction in $|DW|$. We define $\beta_u \triangleq \frac{|\nabla U|^2}{|DW|^2}$ and $\beta_v \triangleq \frac{|\nabla V|^2}{|DW|^2}$. By construction, $\beta \in [0, 1]$ and $\beta_u + \beta_v = 1$. Then (6) can be rewritten as:

$$
u_r = [\beta_u \phi''(s) + \psi(s)\beta_v] U_{NN} + \psi(s) U_{TT}, \qquad (6)
$$

Now we are ready to discuss some desired properties for the function ϕ. This is more complicated than image restoration problems as ϕ is intrinsically a function of both U and V.

- In the presence of small variations in the deformation, ($|DW|$ small implies $|\nabla U|$, $|\nabla V|$ both small), isotropic smoothing is desirable in each individual deformation direction. It is reasonable to require non-trivial smoothing along the tangent direction:

  $$
  \phi'(0) = 0, \quad \text{with} \lim_{s \to 0^+} \psi(s) > 0. \qquad (7)
  $$

  To have isotropic diffusion as $s \to 0^+$ is equivalent to:

  $$
  \lim_{s \to 0^+} \beta_v + \beta_u \frac{\phi''(s)}{\psi(s)} = 1.
  $$

  Together with the fact that $\beta_u + \beta_v = 1$, isotropic diffusion for small deformation implies

  $$
  \lim_{s \to 0^+} \psi(s) = \lim_{s \to 0^+} \phi''(s) > 0. \qquad (8)
  $$

  Once the conditions 7 and 8 are satisfied, the flow (6) for small variation reduces to:

  $$
  u_r \approx \phi''(0) \Delta U.
  $$

  The same analysis holds for v_r.

- In the presence of large variations in deformation (large $|DW|$), it is desirable to diffuse along the discontinuity, but not across it. We need to keep in mind that the level of discontinuity $|DW|$ takes into account deformation in all directions, and the diffusion process in a certain direction (u or v) is decomposed with respect to its own gradient field. In other words, the diffusion process in U direction is the projection of the joint deformation flow onto that direction. To preserve discontinuity, it suffices to annihilate the coefficients of U_{NN} and V_{NN} for large $|DW|$, and assume non-vanishing coefficients for the tangent flow components.

  $$
  \begin{cases}
  \lim_{s \to +\infty} \beta_u \phi''(s) + \psi(s)\beta_v = 0; \\
  \lim_{s \to +\infty} \psi(s) > 0.
  \end{cases}
  $$

If one were to insist on the annihilation of the normal flow for all possible combinations of (β_u, β_v), it would be necessary to require:

$$\lim_{s\to+\infty} \phi''(s) = 0 \quad \text{and} \quad \lim_{s\to+\infty} \psi(s) = 0.$$

On the other hand, if $\beta_u \approx 0$, indicating that the variation in $u-$direction is relatively small, isotropic diffusion in that direction would not result in over-smoothing discontinuity and should be acceptable. With V being the major contributor to the overall discontinuity in $|DW|$, only V_{NN} has to be annihilated. Unfortunately, this again results in a set of incompatible conditions on ϕ:

$$\lim_{s\to+\infty} \phi''(s) \le 0 \quad \text{and} \quad \lim_{s\to+\infty} \psi(s) \ge 0.$$

One possible compromise is to let both terms approach zero as $s \to +\infty$, but at different rates:

$$\begin{cases} \lim_{s\to+\infty} \phi''(s) = \lim_{s\to+\infty} \psi(s) = 0; \\ \lim_{s\to+\infty} \frac{\phi''(s)}{\psi(s)} = 0. \end{cases} \tag{9}$$

Many functions satisfy the above conditions 7,8 and 9, *e.g.*, the hypersurface minimal function. Due to the nonconvex nature of registration problems, we are only interested in finding reasonable local minima in general. When E_d is nonconvex in W, it is not necessary to insist on ϕ being convex.

3. A TEST SETUP

For simplicity, we consider mono-modality registration with L_2 norm as the data fidelity measure, *i.e.*,

$$E_d = \frac{1}{2}\int_\Omega (g(\mathbf{x}) - f(\mathbf{x} + W(\mathbf{x})))^2,$$

and the corresponding variational descent flow is given by:

$$w_d(\mathbf{x}) = (g(\mathbf{x}) - f(\mathbf{x} + W(\mathbf{x}))\nabla f(\mathbf{x} + W(\mathbf{x})).$$

For the preliminary test, we use truncated quadratic [5] as the regularization function:

$$\phi(s,\alpha) = \begin{cases} (\frac{\alpha_0}{\alpha})^2 s^2 & |s| \le \alpha \\ \alpha_0^2 & \text{otherwise.} \end{cases} \tag{10}$$

The disadvantage and benefit of this choice are both obvious. With strict "saturation" behavior above the scale parameter α, it poses a challenge for optimization. Graduated nonconvexification approaches have to be utilized. On the other hand, this formulation provides nice theoretical interpretations. It is natural to introduce a line process [6] which is equivalent to "labeling" the outlier in the robust estimation setting [7].

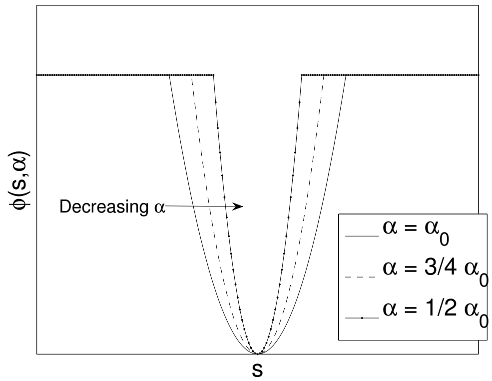

Fig. 1. Truncated quadratic regularization with varying scale.

Notice that (10) also provides a simple recipe to extract singularity set S of $|DW|$ from the estimated W by thresholding at level α:

$$S = \{\mathbf{x} : |DW(\mathbf{x})| > \alpha\}.$$

This is useful if one is interested in extracting motion interfaces.

To alleviate the local minima issue due to nonconvexity, we start with a large initial α. This is equivalent to use conventional Tikhonov regularization of the form $E_r = |\nabla U|^2 + |\nabla V|^2$ as $S = \emptyset$ for α large enough. Then the scale parameter α is gradually decreased till the desired tolerance for discontinuity. To speed up the implementation, a multi-resolution scheme is applied.

4. RESULTS

We apply the setup described in Section 3 to two coronal CT slices obtained from deep inhale and exhale phases. Proposed regularization results in smooth deformation in homogeneous organ (lung, heart and exterior of rib-cage) and correctly preserves motion interfaces on the boundaries between the diaphragm, heart atria, rib cage and the lungs.

5. CONCLUSION

We have proposed a class of regularizations that preserve discontinuities in deformation fields. It applies a robust estimation function to a coordinate-free measure of deformation variation to ensure smoothness in most ROI, yet also allowing for singularities. In the future, we will further investigate numerical aspects of this problem, 3D applications and assess the performance quantitatively.

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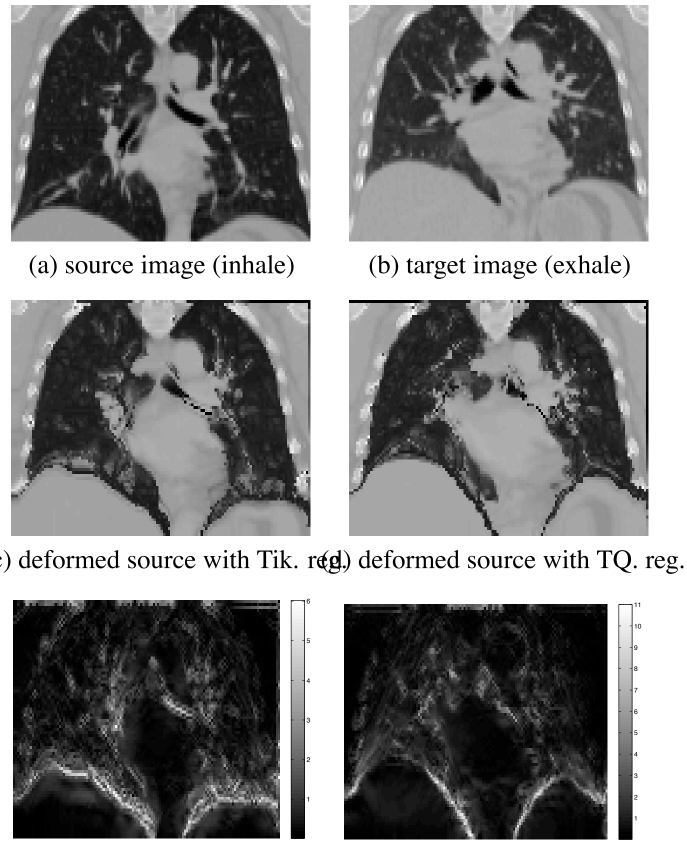
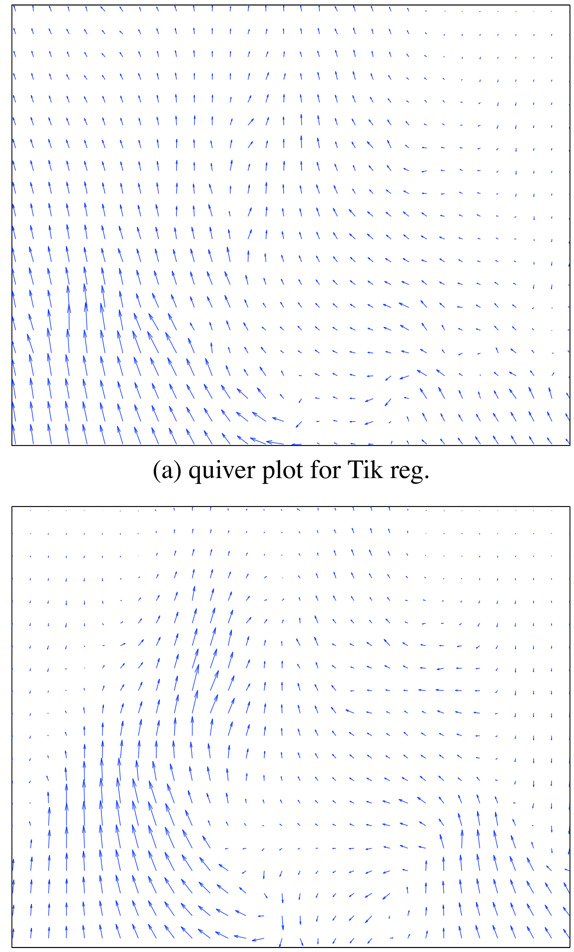


(a) source image (inhale) (b) target image (exhale)

(c) deformed source with Tik. reg. (d) deformed source with TQ. reg.

(e) $|DW|$ with Tik. reg. (f) DW with TQ. reg.

Fig. 2. Registration comparison between Tikhonov (Tik) and Truncated quadratic (TQ) regularizations.



(a) quiver plot for Tik reg.

(a) quiver plot for TQ reg.

Fig. 3. Comparison of deformation fields.

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