Directionally Selective Regularization for Sliding Preserving Medical Image Registration

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***Abstract*—Medical image registration aims to recover the physical deformations between source and target images. As an ill-posed optimizatio problem, regularization energies are often used to guide the registration towards physically desirable resolutions. One challenge in registering volumetric images deformed by respiration-induced motion is to produce motion fields that are globally smooth, yet to allow for sparing physical sliding. In this study, we propose to utilize available partial prior knowledge about the sliding sites as well as their directions. We provide a general recipe for relaxed regularization that incorporates such prior knowledge. The proposed relaxation is both simple and flexible enough to be combined with robust penalty designs that would further improve the registration performance. In a preliminary test, two thoracic CT volumes at end inhale and end exhale were registered under extremely limited partial prior conditions, and the proposed method demonstrated its promise even in this pessimistic scenario.**

I. Introduction

IMAGE registration aims to recover the physical coordinate transformation between source and target observations. This is particularly challenging for medical applications: the underlying deformations are governed by biomechanical properties of the organs, and are very complex - a reasonable description of such motion requires models with high degrees of freedom. With the limited amount of data, estimating such motion is ill-posed and regularization is usually used to guide the optimization towards physically desirable solutions. Smoothness penalty is the most commonly used regularization with demonstrated utility. Unfortunately, sliding along tissue/organ boundaries widely exists, and violates the implicitly assumed homogeneity and isotropy assumptions of most smoothness regularizations.

Several studies [1] [2] of joint segmentation and registration have arisen from various disciplines and applications. In these methods, smooth regions and singularity sets (edges) are devised according to image intensity, and registration aims to align each part respectively. However, sliding motion only occur at a small portion of the intensity edges, and complete independent registration releases all coupling between neighboring organs, which often interacts in specific fashion. Moreover, despite its ability to introduce inhomogeneity in the deformation, segmentation does not provide anisotropic smoothing in the solution.

More recently, sliding preserving regularization energies have been designed so that the corresponding energy-descending variational flows lead to anisotropic smoothing along but not across motion discontinuity boundaries [3] [4]. These studies have demonstrated potential to achieve more physical results, but have revealed numerical instability in 3D registration: 1D sliding singularity may occur as an artefact. This motivated us to study regularization energies where singularity set is restricted to be 2D manifolds. Observing that the sliding sites constitute a proper subset of the intensity edges, and sliding occur *along* the organ boundaries, which can be quite reliably inferred from CT image, the current study takes advantage of such partial prior information to properly relax the regularization according to the potential sliding directions. Intuitively, the set relationship between the sliding sites and the intensity edges provides inhomogeneity in smoothing; while anisotropy is a result of the enforced directional sliding along 2D manifolds with normals in the direction of intensity gradients. A preliminary test was conducted to register two thoracic CT volumes at end inhale and exhale. Even under the most pessimistic inference scenario, the proposed method produced promising results.

II. Background and Existing Work

Let $\Phi:\Omega\rightarrow\Re^3$ denote the deformation vector field, where Ω indicates the region of interest. Given a source image $f:\tilde{\Omega}\rightarrow\Re$ and a target image $g:\Omega\rightarrow\Re$, the goal of image registration is to find an estimate of the deformation vector field $\hat{\Phi}\in\Gamma$ such that

$$g|_{\Omega}\approx f\circ(I+\Phi^*)|_{\Omega}, \tag{1}$$

where Γ is the feasible set of deformations.

In the optimization formulation, we seek $\hat{\Phi}$ that minimizes:

$$E(f,g,\Phi)=E_d(f\circ(I+\Phi),g)+\lambda E_r(f,g;\Phi), \tag{2}$$

where $E_d(\cdot,\cdot)$ measures the image intensity discrepancy between deformed source and target images. The regularization energy E_r guides the solution to exhibit desirable properties, with λ balancing the effect of these two competing energies.

The choice of discrepancy measure depends on the data modality. In general, sum of squared distance is used for calibrated monomodality registration and mutual information for multi-modality registration. Correlation based metric is also commonly applied.

We focus on regularization energy, and list the most relevant regularizations on in Table I[1] .

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[1]The ϕ functions are robust penalties satisfying $\lim_{s\rightarrow+\infty}f''(s)=0$ and $\lim_{s\rightarrow+\infty}f'(s)/s=0$.

TABLE I

COMMON PRIOR FOR DEFORMATION Φ

smoothness [5]	$E_r = \int \|D\Phi\|^2$
topology preservation [6]	$\|D\Phi + I\| > 0$ for all $\mathbf{x} \in \Omega$
volume preservation [7]	$E_r = \int (\|D\nabla\Phi + I\| - 1)^2$
local rigidity [8]	$E_r = \int \eta(\mathbf{x}) \left\|D\nabla\Phi\nabla\Phi^T - I\right\|$
discont. preservation [3]	$E_r = \int \phi(\|D\Phi\|)$
sliding preservation [4]	$E_r = \eta_{\text{div}} \int (\nabla\cdot\Phi)^2 + \eta_{\text{curl}} \int \phi(\nabla \times \Phi)$

III. PROPOSED METHOD

To design a regularization energy E_r that encourages smoothness almost everywhere yet preserves slding, we consider the following intuitive procedures:

1) Approximately localize the sliding surfaces and their directions.
2) Design local regularization that preserves sliding along the surface inferred from the previous step.

To accomplish step (1), we observe that sliding often occur along tissue boundaries. More specifically, the collection of sliding surfaces is a (proper) subset of intensity discontinuities. For x-ray based modalities, such as CT, the intensity boundaries can be inferred quite reliably. In other words, a superset of the potential sliding locations can be estimated accordingly, with their normal direction corresponding to the local intensity gradient directions.

Now consider a voxel at $\mathbf{x}$ that is possibly located on a local sliding surface with normal direction $\mathbf{n}(\mathbf{x})$. There are two possible scenarios based on whether local flow direction information is available.

- When the sliding direction $\mathbf{p}$ is approximately known (*e.g.*, superior-inferior sliding between ribcage and lung caused by respiratory motion), we may determine an orthogonal system $(\mathbf{n},\mathbf{p},\mathbf{q})$ and the corresponding componentwise representation of the local deformation $u = <\mathbf{n},\Phi(\mathbf{x})>$, $v = <\mathbf{p},\Phi(\mathbf{x})>$, $w = <\mathbf{q},\Phi(\mathbf{x})>$ as illustrated in Fig. 1. To preserve sliding in the direction of $\mathbf{p}$ where the sliding surface has normal $\mathbf{n}$, we relax the smoothness regularization in $\frac{\partial}{\partial \mathbf{n}} v$.

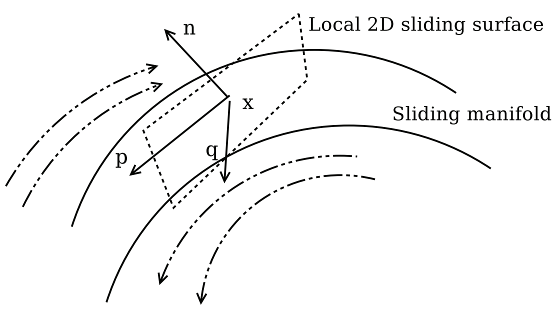


Fig. 1. With information about both sliding surface ($\mathbf{n}$) and sliding direction ($\mathbf{p}$), allow flow along $\mathbf{p}$ to vary "across" the surface (along $\mathbf{n}$).

- When sliding direction is unspecified as in Fig. 2, we allow arbitrary sliding along the predetermined surface: For any $(\mathbf{p},\mathbf{q})$ such that $(\mathbf{n},\mathbf{p},\mathbf{q})$ provides an orthogonal system at $\mathbf{x}$, we generate the componentwise representation (u,v,w) and relax the regularization in $\frac{\partial}{\partial \mathbf{n}} v$ and $\frac{\partial}{\partial \mathbf{n}} w$ simultaneously.

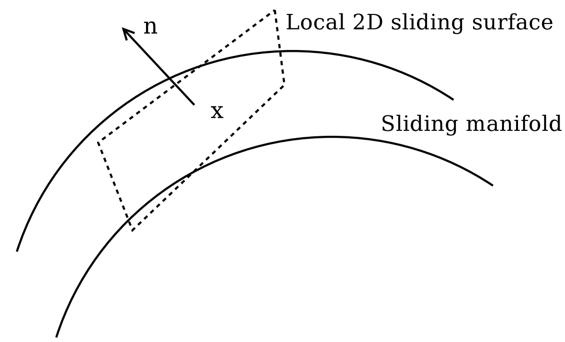


Fig. 2. With information about sliding surface ($\mathbf{n}$), allow flow along surface direction (any $\mathbf{p}$, $\mathbf{q}$) to vary "across" the surface (along $\mathbf{n}$).

In short, we allow the flow to slide by relaxing the directional derivative of the deformation component along flow direction with respect to the (local) sliding manifold's normal direction.

We now make some ramarks and observations to put the current work into context with the existing literatures and discuss its connections with some previous work.

- There exists a clear relationship between the relaxation elements proposed here and the deformation Jacobian structure. At voxel $\mathbf{x}$, the (local) Jacobian represented in coordinate system $(\mathbf{n},\mathbf{p},\mathbf{q})$ reads:

$$D\Phi(\mathbf{x}) = \begin{bmatrix} \frac{\partial}{\partial \mathbf{n}} u & \frac{\partial}{\partial \mathbf{p}} u & \frac{\partial}{\partial \mathbf{q}} u \\ \frac{\partial}{\partial \mathbf{n}} v & \frac{\partial}{\partial \mathbf{p}} v & \frac{\partial}{\partial \mathbf{q}} v \\ \frac{\partial}{\partial \mathbf{n}} w & \frac{\partial}{\partial \mathbf{p}} w & \frac{\partial}{\partial \mathbf{q}} w \end{bmatrix} \qquad (3)$$

  Volume changes are reflected in the divergence of the deformation $(\nabla \cdot \Phi = \mathtt{trace}\{D\Phi\})$; in other words, the volume change is captured with diagonal elements to the first order. As analyzed in [4], all rotational/shear behaviors are captured by the curl component of the flow $\nabla \times \Phi$, which operates on the off-diagonal elements of the Jacobian. In constrast to relaxing all components [3] or the curl component [4] [2] , the current proposal is more selective in the elements to be relaxed. In particular, the relaxation restricted to the directional derivative with respect to the sliding manifold normal eliminates the numerical instability caused by 1D sliding singularity in 3-dimensional registration, as discontinuity is allowed to occur in at most one direction locally.
- The current proposal shares some similarity with joint segmentation and registration work in that the motion boundaries are indirectly inferred from intensity edges. However, there are two distinct differences. First, as a regularization, the spatial over-relaxation by using intensity edge sites in place of *true* sliding sites is implicitly balanced and (hopefully) controlled by the data fidelity term. In other words, sliding will only occur in the solution if it is supported by the intensity fidelity as well as being relaxed by the regularization energy. Furthermore,

[2]The robust function ϕ alleviates the penalty for large values of the operant, which allows the solution to have large values at sparse locations, thus can be regarded as "relaxation" in some sense.

the intensity fidelity energy has a strong impact in steering the solution when there is an intensity edge present (high gradient region), and asserts strong control on the local solution. Therefore, one could argue that relaxing the regularization on the intensity edges - a superset of the true sliding sites - is a viable approach in regularization design, in the absence of stronger prior knowledge. Another distinct feature is the directional selectiveness of the relaxation of the proposed method. Upon identification of a potential sliding site, the relaxation is only applied to the flow components tangential to the local sliding manifold (the v and w components). The tight connecting force across the boundary is ensured as $\frac{\partial}{\partial \mathbf{n}} u$ is always penalized. In contrast, the joint segmentation and registration method needs careful handling of boundary conditions along the segmentation contours. The former behavior is much more desirable from a physiological perspective, as neighboring organs, though may differ in their motion (*e.g.*, sliding), often maintain smooth interaction across their interface. For example, the lung and diaphragm have very distinct attenuation coefficients and their interface can be easily identified in the CT images, yet the interaction between diaphragm and the lung across their boundaries is the major mechanical driving force for respiration - a vanishing $\frac{\partial}{\partial \mathbf{n}} u$ is the proper mathematical property to request.

- The proposed scheme to "selectively" relax regularization to incorporate the sliding preserving properties is independent of the initial regularization scheme. For example, if one starts with the most commonly applied Tikhonov smoothness regularization [5], and chooses a complete relaxation so that the relaxed terms are excluded from the penalty, then the corresponding local regularization at $\mathbf{x}$ is modified to (4).
- The proposed scheme has some natural extensions. Currently, step one provides a binary decision as to whether a local voxel belong to a sliding manifold - it is straight forward to extend this to a probabilistic setting where a soft scale is used for relaxation. In step 2, it is simple to substitute the hard subtraction with robust functions to relax directional derivatives of the sliding flows.

IV. Test Setup and preliminary results

In this test, we registered two CT volumes, obtained from deep inhale and exhale respectively. To fully reflect the possible discrepancy between image intensity discontinuities and motion interfaces, we adopted a pessimistic scheme in determining the sliding sites and applied full relaxed regularization corresponding to nonspecific flow direction to all voxels with nonvanishing local gradient. This means that the regularization energy permits sliding at all spatial locations with any intensity structure, resulting in a very loose superset of the "*true* " sliding sites. This procedure yields a mask that virtually covers the whole region of interest, and only leaves out the air outside the thorax. Despite the poor localization, the local intensity gradient provides directional information for the relaxation and this procedure is well defined.

As discussed in Section III, one often has better prior knowledge or methods to localize organ boundaries and directions in practice. However, this preliminary test provides a performance lower bound on the proposed methodology, and also gives a clear view of its utility that is decoupled from the contributions from other components, such as robust penalty functions.

We compared the proposed method against its original Tikhonov counterpart, both using sum of squared difference as the data discrepancy metric. For a fair comparison, we chose the tradeoff parameters λs such that the intensity discrepancy evaluated at the solutions were similar for both methods. Fig. 5 illustrates typical coronal slices for the source and target images and the deformed intensity images with both methods.

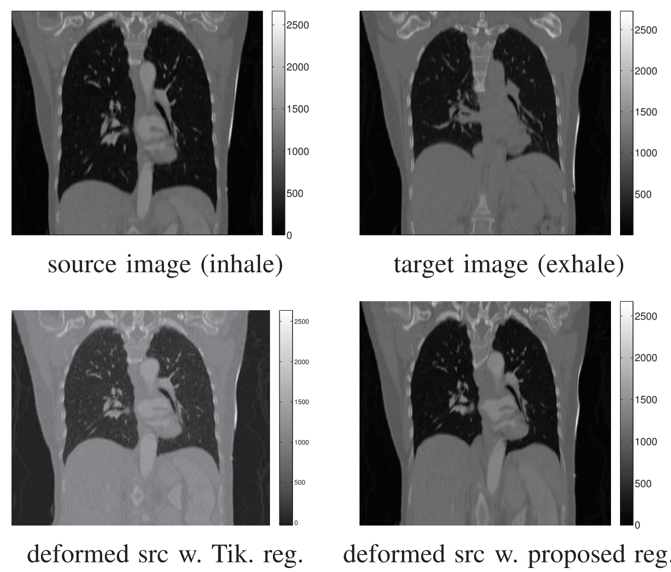
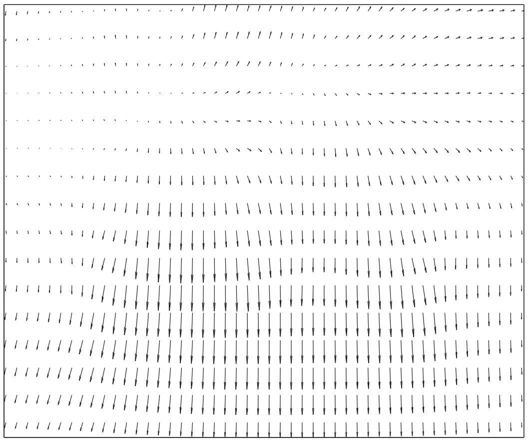


Fig. 3. Comparison of intensity discrepancy between Tikhonov regularization and the proposed relaxation.

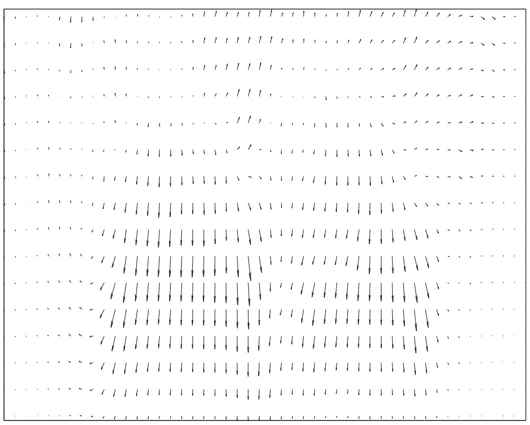
Fig. 4 illustrates (slice views of) the corresponding deformation fields. During CT scan, the patient torso is fixated on the bed while the organs within the ribcage are engaged in involuntary breathing, and one expect to observe small motion outside ribcage and sliding due to respiration along the ribcage. It is quite clear that the proposed method captures such phenomena quite well., while the homogeneous and isotropic smoothing with Tikhonov method leads to unphysical propagation of the flow across the ribcage.

Expecting the respiration-induced sliding to occur mostly in the superior-inferior (SI) direction, we further examine the value of the SI component of the estimated motion field in Fig. 5. It can be observed that the proposed method manages to capture the sliding along the ribcage as well as the motion interface between the heart and lung, while the Tikhonov method fails. On the other hand, the proposed regularization still has much room for improvement: the regularization seems to be insufficient to it seems to lack enough smoothing power. This could be due to the overrelaxation in our setting and will be topic of further investigation.

$$E_r(\mathbf{x}) = \begin{cases} \|D\Phi\|^2_{\text{Frob}} & \text{if } \mathbf{x} \notin \text{sliding manifold} \\ \|D\Phi\|^2_{Frob} - (\frac{\partial}{\partial \mathbf{n}} v)^2 & \mathbf{x} \in \text{sliding manifold with specified flow direction } \mathbf{p} \\ \|D\Phi\|^2_{Frob} - (\frac{\partial}{\partial \mathbf{n}} v)^2 - (\frac{\partial}{\partial \mathbf{n}} w)^2 & \mathbf{x} \in \text{sliding manifold with unspecified flow direction} \end{cases} \quad (4)$$

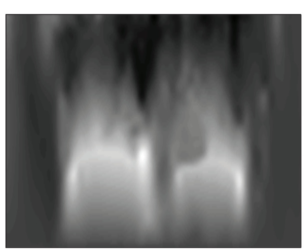


Tikhonov Reg.



with proposed relaxation.

Fig. 4. Comparison of deformation fields (in sliced quiver plot) between Tikhonov regularization and the proposed relaxation.



sliced view of SI def. for TiK reg. sliced view of SI def. for proposed reg.

Fig. 5. Test result of proposed method.

V. Summary

We have proposed a method to systematically relax any given smoothness regularization energy to preserve sliding interfaces. It infers a superset of the sliding interface locations from image intensity edges and determines the normal direction of the sliding interface from the intensity gradient. Depending on the availability of sliding direction information, the proposed method either relaxes a sub-direction of sliding along the interface or relaxes all free sliding following the interface. A preliminary test with volumetric CT data has shown promising results. We will further investigate (1) probabilistic sliding localization schemes rather than binary set assignment for more regularizing power; (2) methods for mild regularization for relaxed components for improved stability. We will further validate the performance of the proposed method with more clinical data.

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References

[1] A. Yezzi, L. Zllei, and T. Kapur, "A variational framework for integrating segmentation and registration through active contours," *Med. Im. Anal.*, vol. 7, no. 2, pp. 171–85, Jun. 2003.

[2] M. Droske and M. Rumpf, "Multiscale joint segmentation and registration of image morphology," *IEEE Trans. Patt. Anal. Mech. Intell.*, vol. 29, no. 12, pp. 2181–2194, Dec. 2007.

[3] D. Ruan, J. A. Fessler, and S. Esedoglu, "Discontinuity preserving regularization for modeling sliding in medical image registration," in *Proc. IEEE Nuc. Sci. Symp. Med. Im. Conf.*, 2008, pp. 5304–5308.

[4] D. Ruan, S. Esedoglu, and J. A. Fessler, "Discriminative sliding preserving regularization in medical image registration," in *Int. Sym. Bio. Im.*, 2009.

[5] B. Horn and B. G. Schunck, "Determining optical flow," *Artif. Intell.*, vol. 18, no. 1-3, pp. 185–203, Aug. 1981.

[6] B. Karacali and C. Davatzikos, "Estimating topology preserving and smooth displacement fields," *IEEE Trans. Med. Imag.*, vol. 23, no. 7, pp. 868–80, Jul. 2004.

[7] T. Rohlfing, C. R. Maurer, D. A. Bluemke, and M. A. Jacobs, "Volume-preserving nonrigid registration of MR breast images using free-form deformation with an incompressibility constraint," *IEEE Trans. Med. Imag.*, vol. 22, no. 6, pp. 730–741, Jun. 2003.

[8] D. Ruan, J. A. Fessler, M. Roberson, J. Balter, and M. Kessler, "Nonrigid registration using regularization that accommodates local tissue rigidity," in *Proc. SPIE 6144, Medical Imaging 2006: Image Proc.*, 2006, p. 614412.