

A linear implementation of dose-volume constraints for multi-criteria optimization

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Abstract— Dose-volume constraints are widely used in inverse planning for radiation therapy. Given the intricate combinatorial nature of the problem, existing approaches suffer from long runtimes, insufficient approximation of the constraints, or the necessity to specify reasonable starting values a priori. This can be problematic, particularly when planning is considered as a multi-criteria optimization problem. We present a new method to handle dose-volume constraints during planning for robotic radiosurgery. Taking into account the typically very conformal nature of the resulting dose distributions, we specify the constraints on a small subset of points instead of the full volume. We show how this allows for an effective relaxation of the problem. Results for a prostate case indicate that the proposed method leads to good approximations of the dose-volume constraints and is independent of the optimization objective.

Keywords— dose-volume constraints, treatment planning, optimization, radiosurgery, CyberKnife

I. INTRODUCTION

While it is debatable whether dose-volume constraints (DVC) sufficiently reflect the actual biological effect of radiation [1,2], they remain an important tool for assessment of treatment plans in clinical practice. However, when DVC are included in inverse planning, the resulting optimization problem is non-convex and multiple minima may exist [3,4]. The severity of the problem will depend on the complexity of the considered cases [5].

For gradient based optimization, penalization schemes have been successfully used to implement DVC [6,7], i.e., there is generally no guarantee of achieving the global optimum. In contrast, mixed integer programming (MIP) will find the global optimum [8]. However, solving a MIP is typically computationally expensive and therefore approaches which use a coarse set of integer constraints have been proposed [9]. One way to completely avoid integer constraints is to apply a bound on the mean dose above a threshold [10]. While the resulting optimization problem can be efficiently solved using linear or quadratic programming, there is no guarantee that the actual DVC is fulfilled. Moreover, in order to specify a constraint on the mean dose it would be necessary to know this value a priori. Generally,

this value can only be estimated from a given dose distribution. Similarly, iterative approaches depend on a known dose distribution [11]. Yet another approach to handle DVC in constrained optimization is to fix an upper bound on each voxel depending on its spatial relation to the planning target volume (PTV) [12].

The latter approach may be less suitable when the isodose surfaces do not follow the target shape. For example, the lower isodose lines in intensity modulated radiation therapy (IMRT) typically exhibit ‘dose fingers’, i.e., the dose follows the beam orientation. Interestingly, this is not generally the case for robotic radiosurgery, where a much larger number of beams is used within a large non-coplanar workspace. Therefore, in general the dose distribution is conformal to the PTV shape even at low isodose levels. We propose a variant of the spatial constraint method that uses an upper bound on a small subset of voxels only. Moreover, we show how this implementation of DVC can be relaxed to yield a better match to the dose volume histogram (DVH) shape specified by the DVC and also improved plan quality. In particular, the method allows a priori specification of DVC without knowledge of the dose distribution, is independent of the optimization objective, and allows implementation of inverse planning as a linear optimization problem suitable for multi-criteria optimization.

II. MATERIAL AND METHODS

We study robotic beam delivery, where a large number of treatment beams are delivered from different non-coplanar directions [13]. Each beam is weighted independently and the inverse planning problem is a special case of direct aperture optimization (DAO) with circular apertures. Initially a large set of candidate beams is heuristically sampled and beam weights are computed using a step-wise multi criteria approach based on linear programming [14].

Consider a DVC such that the dose in an organ-at-risk (OAR) may exceed the dose d for a maximum volume v , then the DVC is implemented as follows. First, the OAR is partitioned into a high dose and low dose region, where the volume of the high dose region is equal to v . Second, a set S of points on the surface separating the two regions is sam-

pled. Third, constraints on the dose in the sampled points are added to the optimization problem. As the isodose surfaces tend to resemble the PTV shape, we compute the surface by growing the PTV isotropically until the OAR subvolume enclosed within this surface is equal to v . The set S is defined as a sample of points on the portion of this surface within the OAR. Hence, for perfectly conformal dose distributions and exactly fulfilled DVC, all points in S will have a dose equal to d . If the surface is sufficiently large, i.e., such that all beams intersect the surface, we can assume that all OAR voxels on the far side of the surface will have a dose smaller than d . Therefore, constraining the points in S to a maximum dose d will effectively enforce the DVC. We refer to this implementation as strict-DVC.

While the proposed approach is straight forward to implement, the assumption of perfectly conformal isodose surfaces is generally over-restrictive, i.e., some parts of the high dose region will actually receive a dose below the threshold. In this case the DVC is still satisfied as it is an upper bound but the resulting plan quality with respect to other criteria will be sub-optimal because the solution is over-constrained.

The parts of the high-dose region that receive a dose below the threshold d typically correspond to a subset of S having a dose below d . Likewise, we can allow a subset of S to exceed d and hence to have some volume in the low dose region exceed d . This trade-off is acceptable as long as the DVC is still fulfilled. To allow for this trade-off, we constrain the mean dose over all points in S instead of placing a fixed upper bound on each point in S . Essentially this approach allows ‘hotspots’ along the surface so long as they are offset by corresponding ‘coldspots’. The resulting mean constraint is further limited by only being applied to doses greater than $d-T$, where T is a relaxation parameter which effectively limits the magnitude of ‘hotspots’ along S . We refer to this implementation as relaxed-DVC.

The method has been applied to prostate and lung cases and is illustrated here in the context of prostate treatment. The target prescription dose, covering at least 95% of the PTV, was 36.25 Gy, with the maximum dose in PTV, rectum, and bladder constrained at 41, 37, and 39 Gy, respectively. Further constraints on the maximum dose in the penile bulb (30 Gy), the seminal vesicles (40 Gy), left and right femur (18 Gy) as well as on a set of shell structures (isotropically expanded surfaces surrounding the PTV at varying distances) were in place. DVC restricting the volume receiving more than 18 Gy to 33 % of the respective OAR volume were set for rectum and bladder. The DVC-relaxation was applied to the top 3 % of the points in S (i.e. $T = 0.5$ Gy).

To study the robustness of the proposed method with respect to different clinical objectives we considered three

optimization scenarios. First, the objective was to generate a very homogeneous PTV dose by minimizing the maximum dose in the prostate. Second, we optimized with respect to conformality. Third, the total beam weight was minimized. Each optimization objective was run without DVC, with strict-DVC, and with relaxed-DVC.

III. RESULTS

Tables 1 – 3 summarize the optimization results for the combinations of DVC method and optimization objective. As expected, the best objective values are realized without DVC. When the maximum dose in the PTV is minimized to obtain a more homogeneous dose distribution, the relaxed DVC leads to a lower value than the strict-DVC (Table 1, first column). Likewise, if the maximum dose in a shell is minimized as a surrogate for the dose gradient and conformality, the relaxed-DVC allows for a more conformal dose distribution. In fact, the resulting maximum dose in the shell is almost the same as that achieved without DVC (Table 2, second column). The most substantial difference is seen when the total beam weight is minimized. Table 3 shows that the result for the relaxed-DVC is 15418 MU compared to 14327 MU without DVC. In contrast, the strict-DVC the method results in 18768 MU. Furthermore, the number of treatment beams also remains higher when the strict-DVC is in place.

Table 1 Results after optimizing homogeneity

Method	Max PTV (Gy)	Max SHELL (Gy)	# beams	# MU
no-DVC	37.56	36.27	276	30000
strict-DVC	38.32	37.59	238	30000
relaxed-DVC	37.81	36.34	262	30000

Table 2 Results after optimizing conformality

Method	Max PTV (Gy)	Max SHELL (Gy)	# beams	# MU
no-DVC	4100	3191	223	30000
strict-DVC	4100	3212	221	30000
relaxed-DVC	4100	3192	223	30000

Table 3 Results after minimizing the total beam weight

Method	Max PTV (Gy)	Max SHELL (Gy)	# beams	# MU
no-DVC	4100	3701	95	14327
strict-DVC	4100	3800	125	18768
relaxed-DVC	4100	3621	101	15418

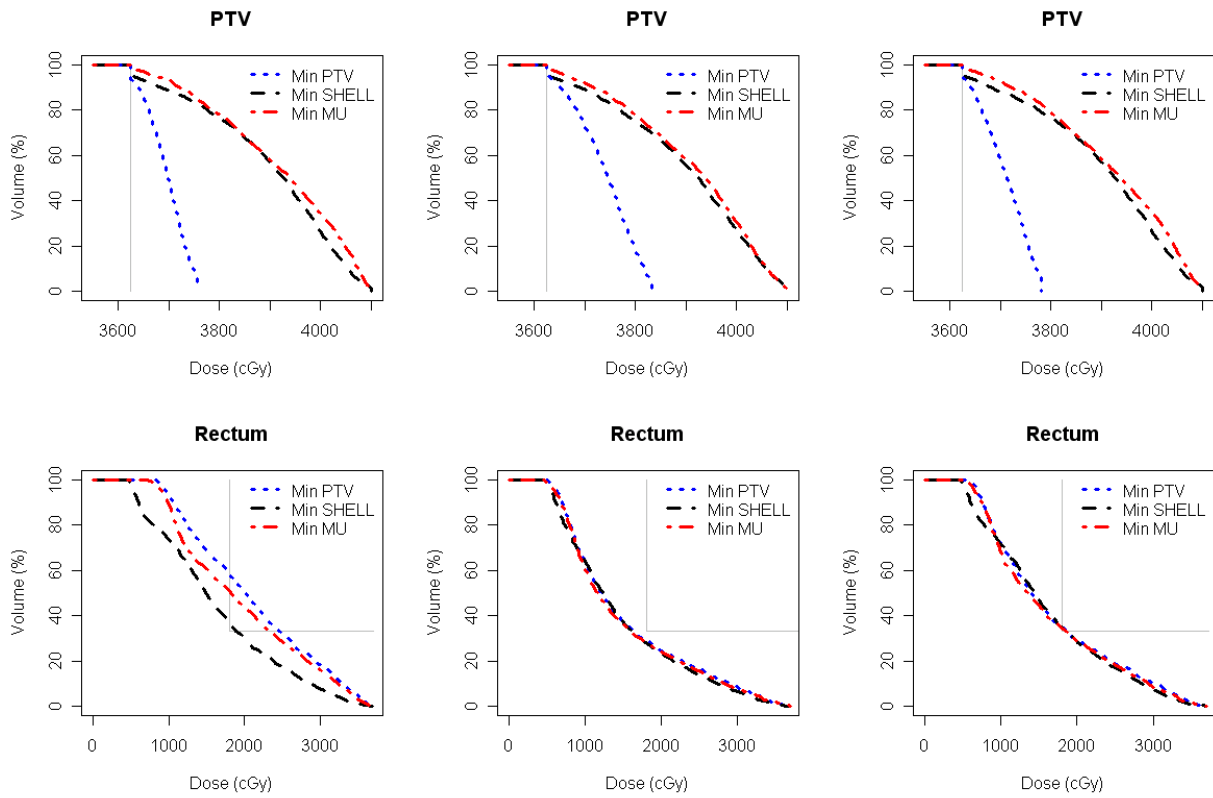


Fig. 1 DVHs for PTV and rectum, with no-DVC (left), strict-DVC (center), and relaxed-DVC (right).

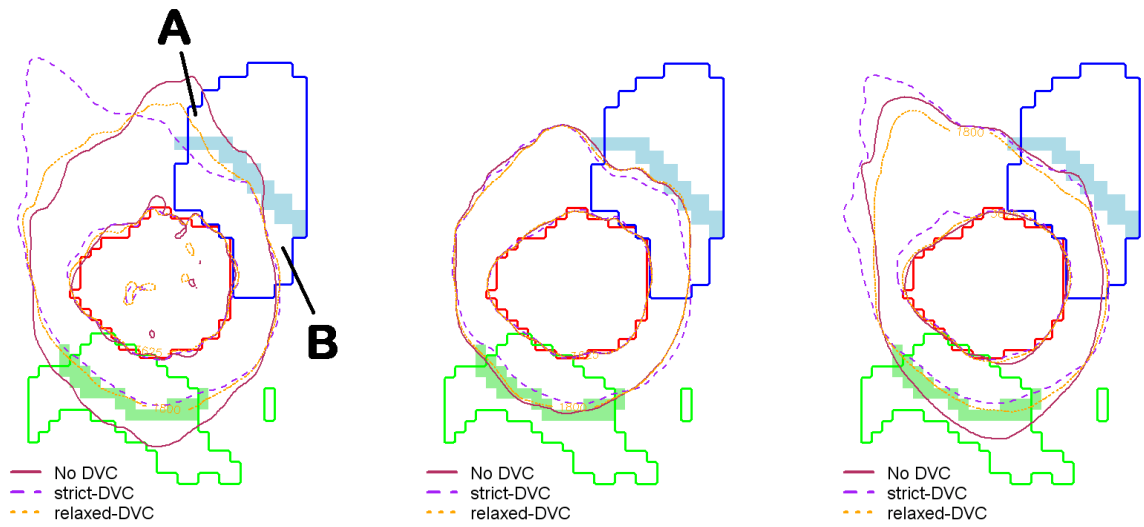


Fig. 2 A sagittal slice showing PTV (red), rectum (green), bladder (blue), and the two DVC surfaces (light green, light blue), as well as 18 Gy and 36.25 Gy isodose contours for min PTV (left), min SHELL (center), and min MU (right) optimization objectives.

The PTV dose distributions are illustrated in the first row of Figure 1. More interestingly, the first DVH in the second row of Figure 1 shows that the dose distribution in the rectum is not acceptable for any optimization objective when no DVC is in place. The second DVH in that row illustrates that the strict-DVC is over-restrictive. The relaxed-DVC leads to a good approximation of the desired dose distribution for all three optimization objectives. Results for the bladder are similar.

The difference between strict and relaxed-DVC is also visualized in Figure 2. When the maximum dose in the PTV is minimized, the unrestricted isodose line has a slightly non-conformal shape towards the bladder. While the strict-DVC forces the 18 Gy iso-dose line onto the PTV side of the surface structure (light blue), the relaxed-DVC allows to compensate the higher doses in region A by lower doses in region B. Note that the isodoses are fairly conformal for all three optimization objectives.

IV. CONCLUSIONS

The results indicate that the proposed approach can be used to model DVC for stepwise multi-criteria optimization of robotic radiosurgery. Advantages of the relaxed-DVC method include a straightforward implementation as pure constraints, independence from different optimization objectives, and fast optimization as linear program. Existing linear approximations of DVC use the objective function or require a priori knowledge of the dose distribution [10,11], which makes it impractical to balance the trade-off between the DVC and other clinical objectives.

Clearly, the presented method is only an approximation. One limitation lies in the surface generation. When only a very small subvolume of an OAR is subject to the DVC, the surface will be small and not all beams hitting the OAR will also intersect the surface. Therefore, the assumption that the surface partitions the OAR into high and low dose region may not hold. However, this can be easily detected and in such a case it would be reasonable to specify the location of the hot spot directly [15].

Another limitation is the relaxation, i.e., there is no principle rule to determine the amount of relaxation suitable for a given case. So far, small relaxations in the order of 3-5 % of the dose threshold have consistently led to good results. Furthermore, the relaxation parameter itself can be considered as one objective, i.e., while maintaining all other constraints, a variable expressing the relaxation is minimized. Hence, the trade-off between various other clinical goals and the objective to strictly fulfill the DVC can be analyzed in a systematic way.

Finally, in the context of stepwise multi-criteria optimization the method can be easily adapted to also provide an optimization objective (of the form; minimize the dose received by X% of an OAR towards a goal of d), and then set the result as a constraint for subsequent optimization steps.

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